Advanced Analysis: Correlation of EVs and Design Efficiency

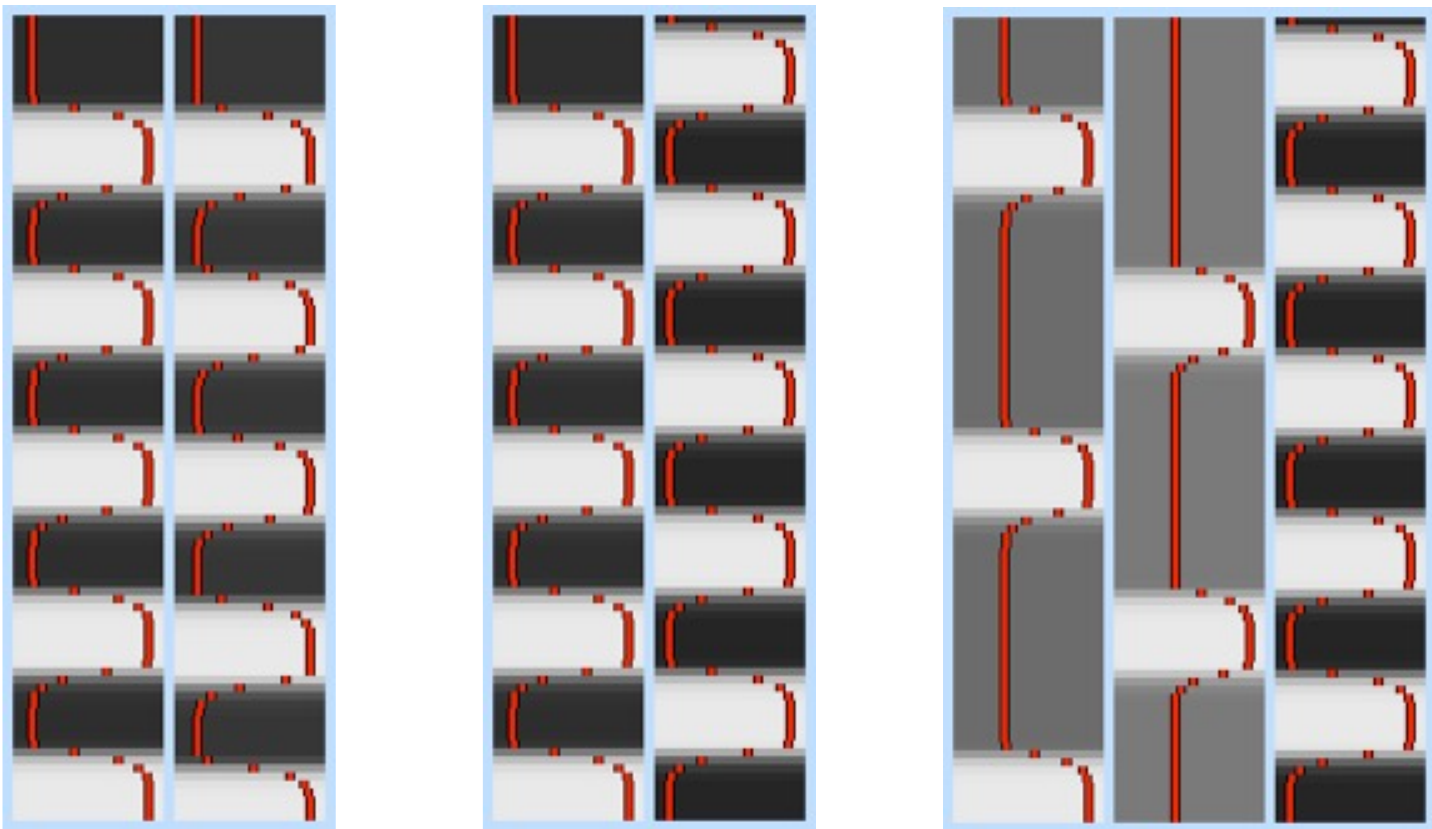
Correlation of EVs

- Correlated EVs are relatively common, but **strong correlation is a problem** in either first-level or group-level designs.

- When EVs are correlated, it is the ***unique contribution*** from each EV that determines the model's fit to the data and the statistics.

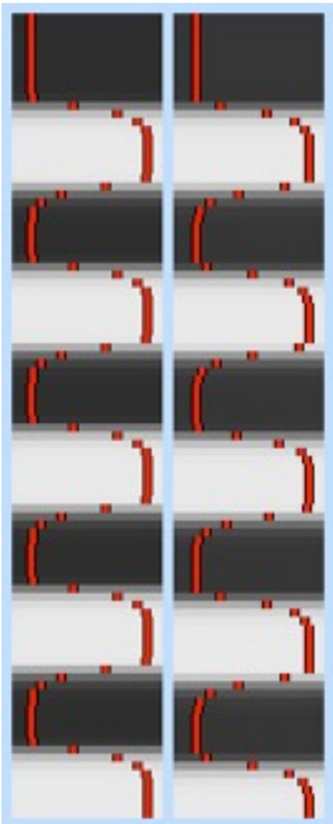
- Start by looking at first-level examples:
  - correlation and rank deficiency
  - design efficiency tool

Correlation of EVs: First-level designs



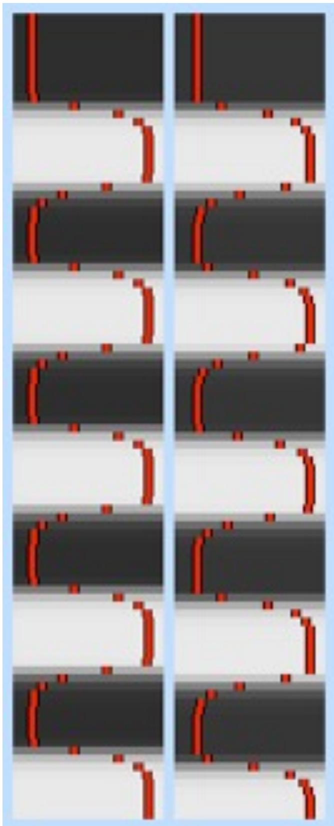
Design Matrix Rank Deficiency

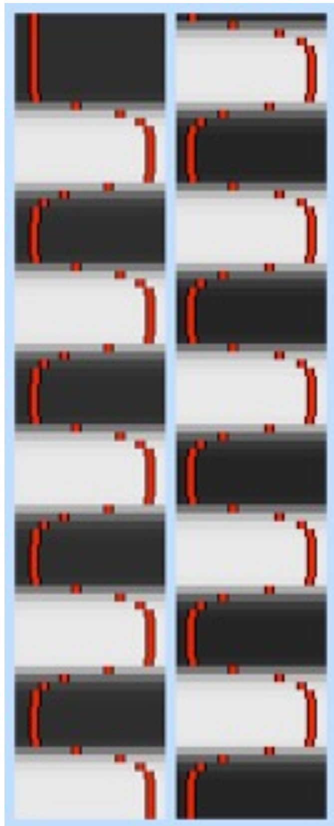
- A design matrix is rank deficient when a linear combination of EVs is exactly zero
  - Model can fit exactly the same signal in multiple ways!
- e.g. visual and tactile stimulation occurs at very similar times, so it is not possible to separate the responses!



Design Matrix Rank Deficiency

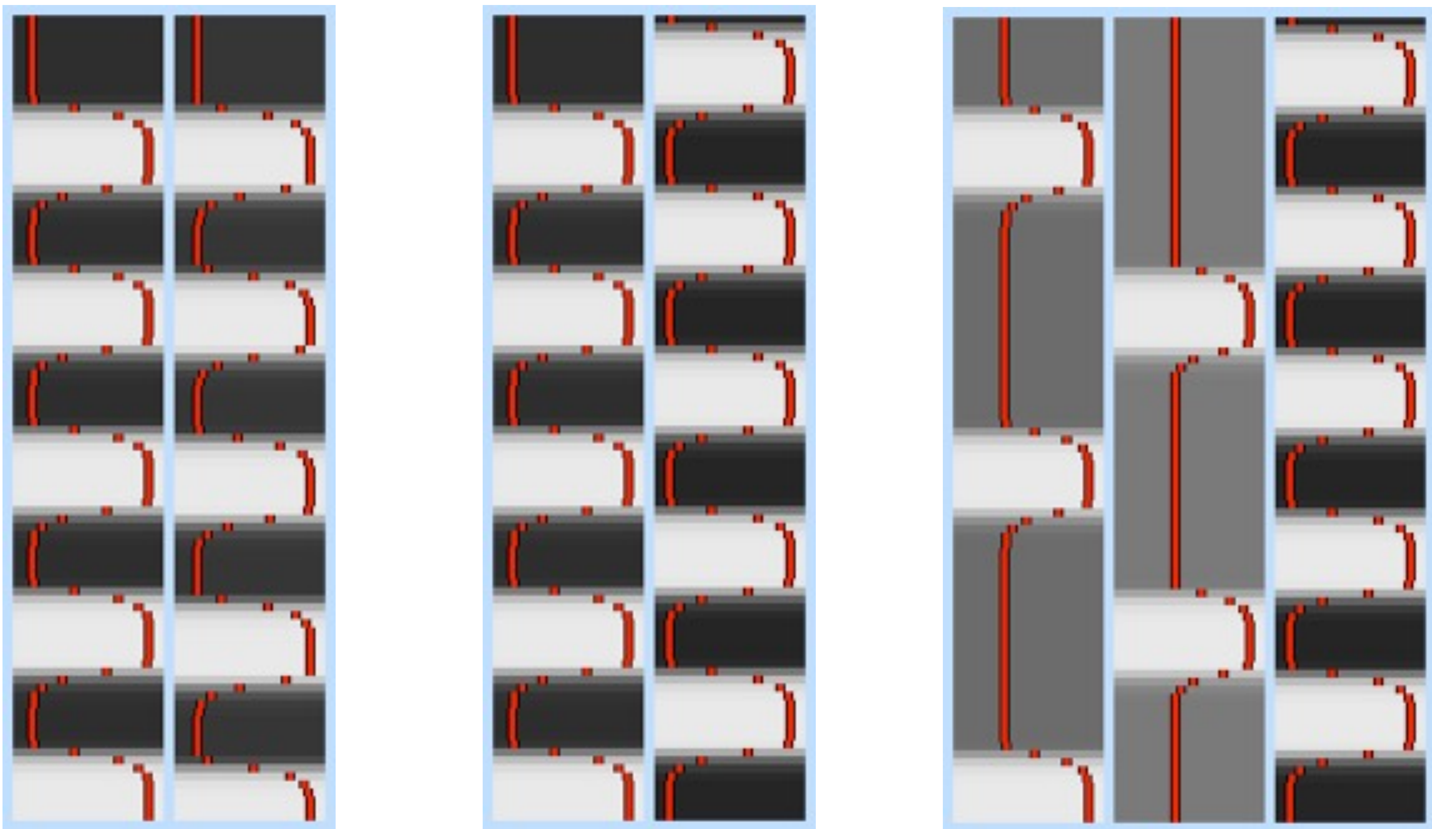
- A design matrix is rank deficient when a linear combination of EVs is exactly zero
  - Model can fit exactly the same signal in multiple ways!
- e.g. visual and tactile stimulations are exactly opposed (so no baseline)





Design Matrix Rank Deficiency

- A design matrix is rank deficient when a linear combination of EVs is exactly zero
  - Model can fit exactly the same signal in multiple ways!
- e.g. modelling visual, tactile, and rest (the last one is effectively baseline and shouldn’t be modelled in FSL)



Close to Rank Deficient Design Matrices

- **Good News:** The statistics always take care of being close to rank deficient

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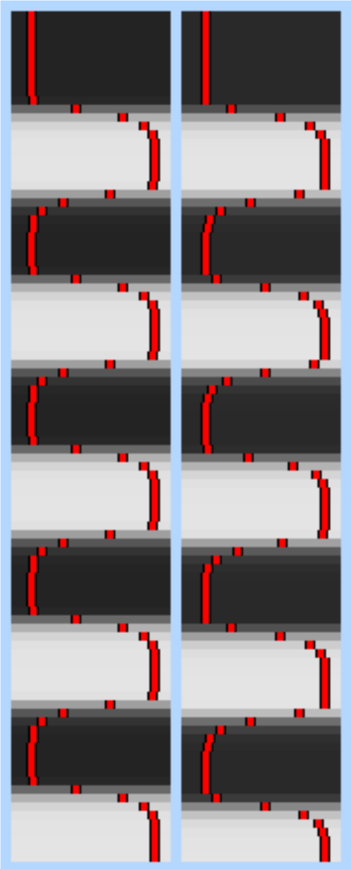
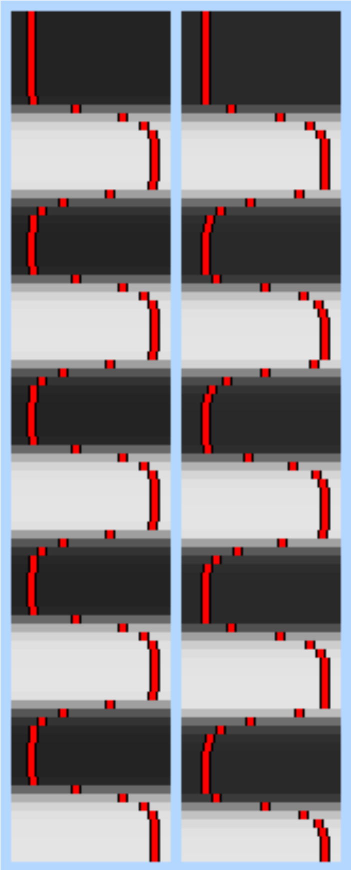
- **Bad News:** the ignorant experimenter may have found no significant effect, because:
  a) Effect size was too small.
  b) Being close to rank deficient meant finding an effect would have required a HUGE effect size
     e.g. may need a lot of data to determine how two EVs with very similar timings best combine to explain the data.

When do we have a problem?



- Depends on SNR, and **crucially** the contrasts we are interested in:
- [1 -1] e.g. vis-tact??
- [1 1] e.g. average response??
- [1 0] or [0 1] ?? e.g. visual? or tactile?

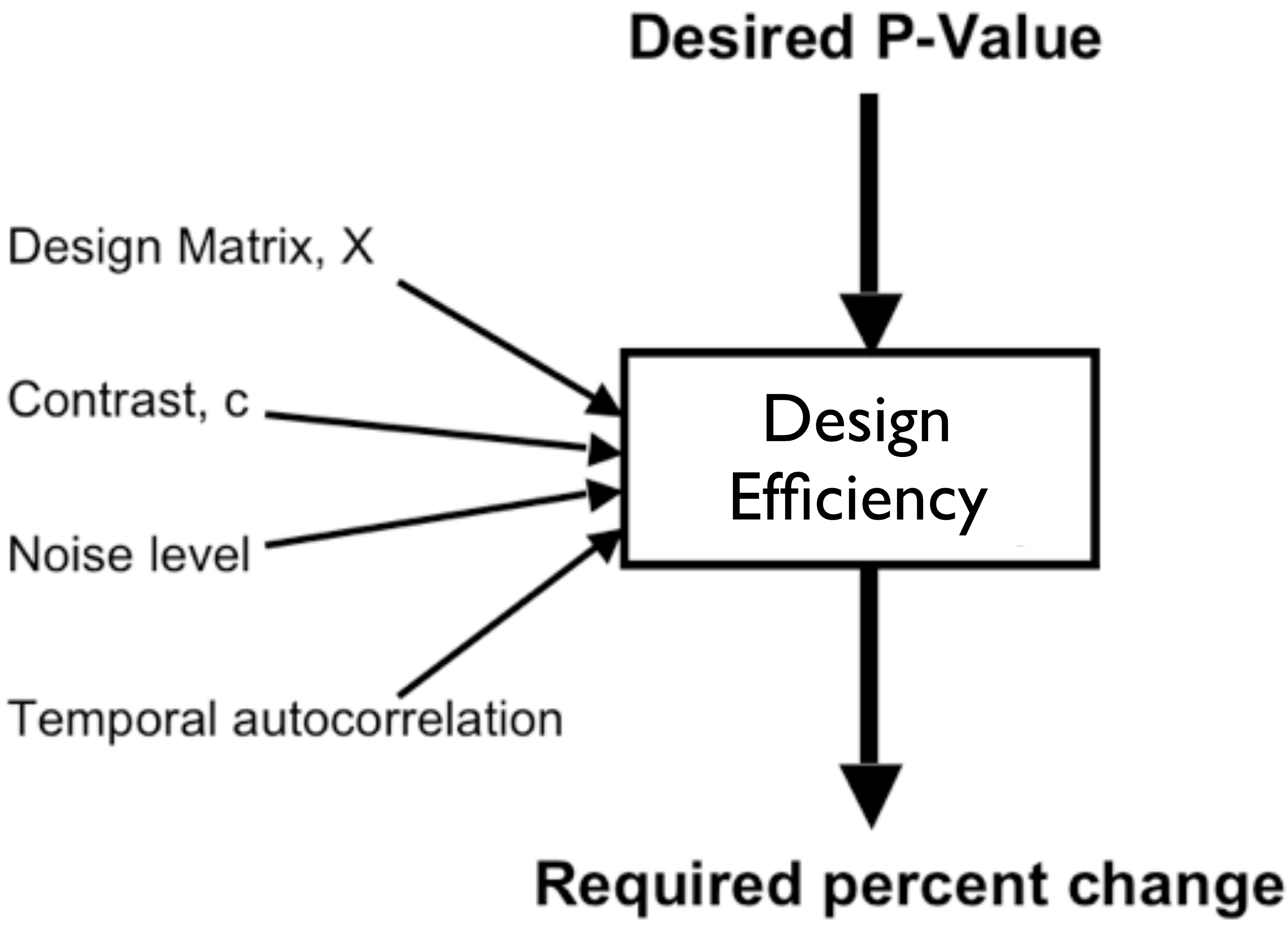
When do we have a problem?



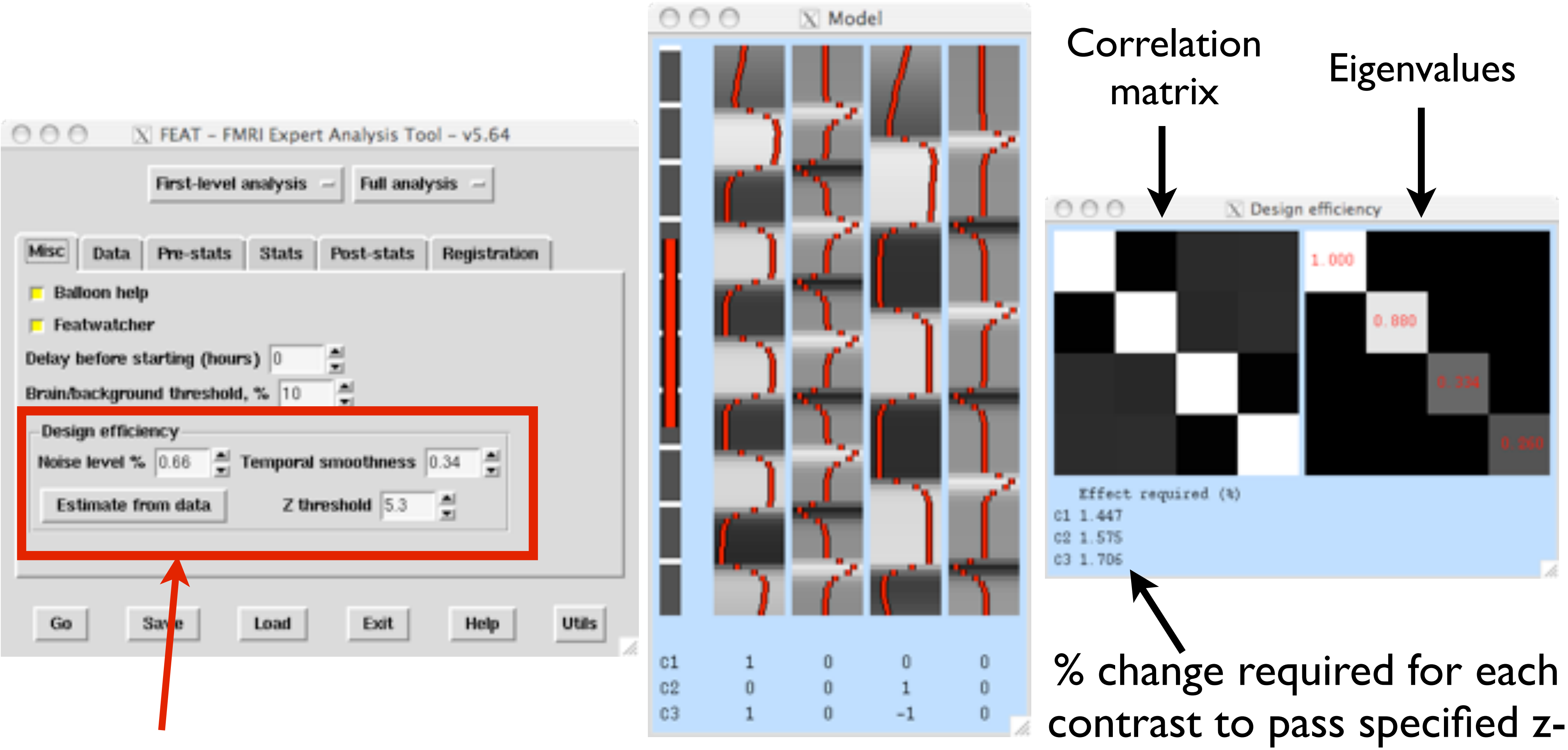
- Depends on SNR, and **crucially** the contrasts we are interested in:

- [1 -1] e.g. vis-tact??
  - no chance:
- [1 1] e.g. average response??
  - no problems:
- [1 0] or [0 1] ?? e.g. visual? or tactile?
  - no chance:

Design Efficiency



Design Efficiency

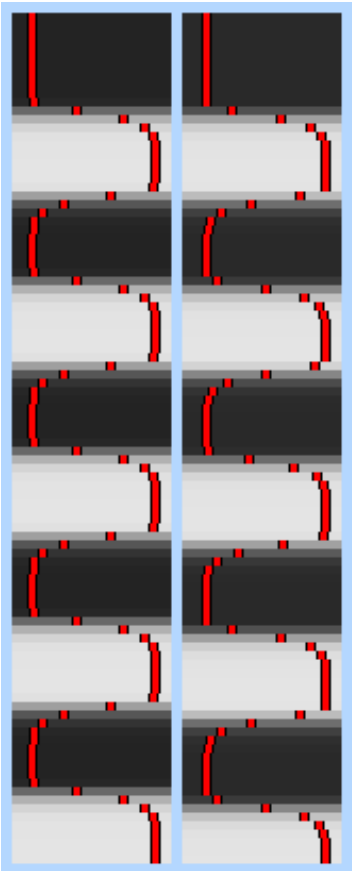


% change required for each contrast to pass specified z-threshold

Settings for design efficiency calculations

These are the most useful!

When do we have a problem?



- Depends on SNR, and **crucially** the contrasts we are interested in:

- [1 -1] e.g. vis-tact?? **<u>Effect size required</u>**
  - no chance: **5.3%**
- [1 1] e.g. average response??
  - no problems: **0.84%**
- [1 0] or [0 1] ?? e.g. visual? or tactile?
  - no chance: **5.3%**

Case Study: Correlated EVs

Scenario:

Investigating whether there is a relationship between a patient's disease/behavioural scores and their BOLD responses

Problem:

Different scores are likely to be strongly correlated.

Which regions' responses correlate with disease scores but not age?

Solutions:

Combination of F-tests and t-tests

Correlations, Covariates & Corrections

- Consider a case example:
  - Disease Duration (DD) + age (demeaned)
    - where we want to ‘correct’ for age

Correlations, Covariates & Corrections

- Consider a case example:
  - Disease Duration (DD) + age (demeaned)
    - where we want to 'correct' for age
  - If there is correlation between DD and age then it becomes tricky
  - One option is orthogonalisation of DD and age …

Orthogonalisation

Orthogonalisation

DON'T DO IT!

A better alternative to orthogonalisation

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  - Disease Duration (DD) + age (demeaned)
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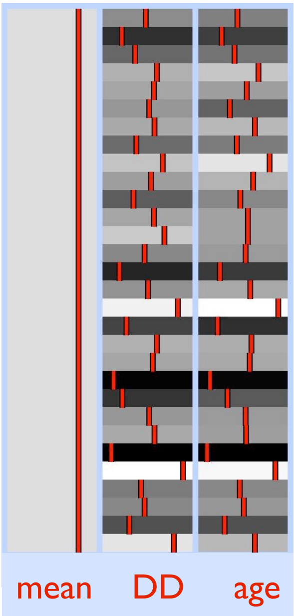
t-test



[0 1 0]

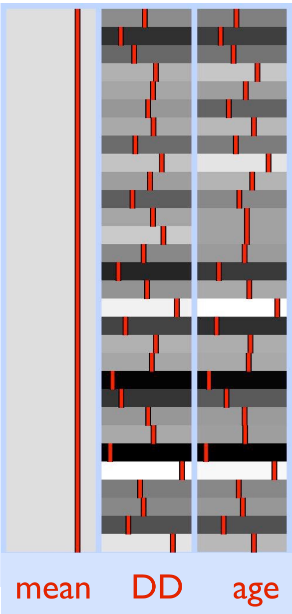
A t-test for a single EV is determined only by variability in BOLD signal that *cannot* be accounted for by other EVs.

This is a ***conservative*** result: only when DD can *uniquely* explain the measurements will there be a significant result.

A better alternative to orthogonalisation

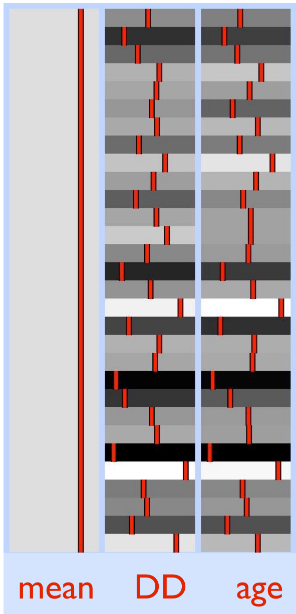
- Consider a case example:
  - Disease Duration (DD) + age (demeaned)
    - where we want to 'correct' for age

t-test



[0 1 0]

F-test



[1 0 0]

[0 1 0]

[0 0 1]

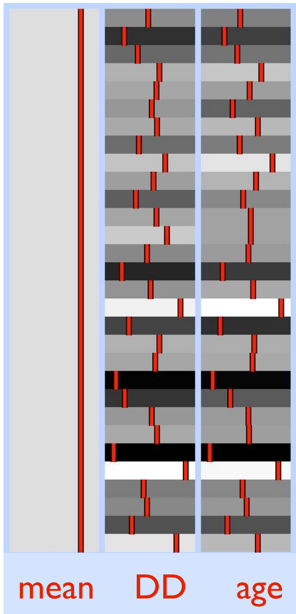
A better alternative to orthogonalisation

- Consider a case example:
  - Disease Duration (DD) + age (demeaned)
    - where we want to ‘correct’ for age

F-test

An F-test finds regions where signal can be explained by *any combination* of EVs.

Will show significant results where either DD or age or both can explain the measurements.



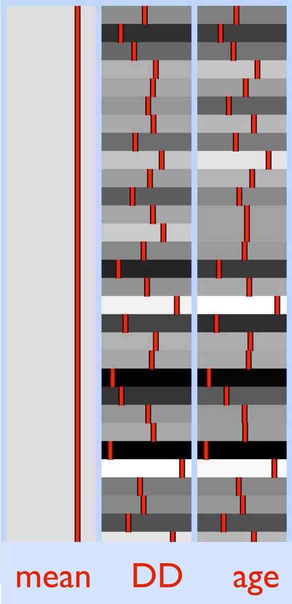
[1 0 0]

[0 1 0]

[0 0 1]

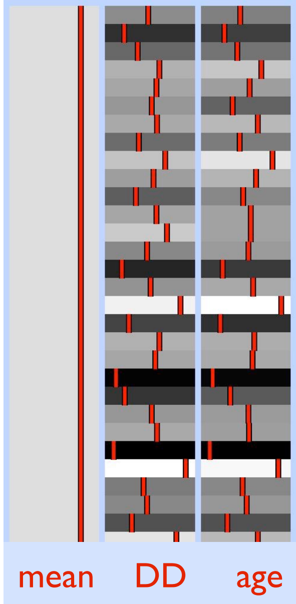
A better alternative to orthogonalisation

t-test



[0 1 0]

F-test



[1 0 0]

[0 1 0]

[0 0 1]

<u>Results (a fairly typical example with strong correlation)</u>:

Not significant (t-test) Significant (F-test)

<u>Interpretation</u>: Significant correlation with *both* DD and age, but cannot separate the effects as they are too highly correlated and the response to unique portions (if any) are too weak.

<u>Follow on</u>: one way to (potentially) separate the effects would be to recruit new subjects such that DD and age were less correlated (need more data to go beyond the above interpretation).

Advanced Analysis: Correlated EVs

Summary:

- Correlation of EVs makes it difficult for the GLM to assign unique contributions and often leads to no significant results
- Extreme correlation gives rank deficiency
- Problem of correlation depends on the contrast
- Design efficiency gives required % BOLD change to get a significant result *per contrast* (like power calc.)
- Can also get info about where correlations are
- Orthogonalisation: DON'T DO IT!
- In practice consider F-tests for combined explanatory results as well a t-test (unique contributions)
- Try to break correlations through planning/recruitment